AN OPTIMIZATION MODEL FOR RIVER BASIN WATER RESOURCES ALLOCATION WITH RESERVOIRE REGULATION

YANGBO CHEN
Department of Water Resources and Environment, Sun Yat-Sen University, 135 Xingangxi Road, Guangzhou, Guangdong Province 510275, China

BIQIU ZENG
Department of Water Resources and Environment, Sun Yat-Sen University, 135 Xingangxi Road, Guangzhou, Guangdong Province 510275, China

This paper presents an optimization model for river basin water resources allocation with the reservoir regulation in the upper stream. The optimal objective is to minimize the loss caused by water shortage, while the constraints include water balance, flood control, reservoir storage, navigation, turbine discharge and power generation. The Dynamic Programming was employed to solve the model. Xizhijiang River Basin in Southern China was investigated.

INTRODUCTION

Due to the rapid population growth and economic development, the world water demand increases sharply, while on the other hand, the clean water decreases as water quality deterioration and climate variation. Current days, water shortage is a global issue affecting the regional sustainable development. To rationally allocate water resources is a key measurement to deal with water resources shortage.

For a river basin, there is usually built regulating reservoir in the upper stream that can regulate the river discharge in the dry season and dry year. Unfortunately many reservoirs operate for the purposes of power generation, thus not taking the potential for regulating water resources allocation in the basin scale. It is needed to rationally make the reservoir operation scheme to deal with the increasing water shortage.

There are three different methods to make reservoir operation schemes, namely Deterministic Optimization (DO), Explicit Stochastic Optimization (ESO) and Implicit Stochastic Optimization (ISO), for solving the water resources planning problem. Deterministic Optimization method is a good choose. Dynamic Programming (DP)[8] is a classic method of DO, other methods include Dynamic Programming Successive Approximation (Korsak, 1970)[6], Discrete Differential Dynamic Programming (Heidari, 1971)[4], Binary State DP (Ozden, 1984)[7], LP-DP (Becker, 1974)[1], Network Flow Optimization (Jensen, 1978)[5], Hierarchical Analysis (Haimes, 1977)[3].

This paper presents an optimization model for river basin rational water resources allocation by regulating the reservoir in the upper stream, so to increase the water resources in dry season and dry year. The model optimal objective is to minimum the loss caused by water shortage, while the constraints include water balance, flood control,
reservoir storage, navigation, turbine discharge and power generation, and the Dynamic Programming was employed to solve the model. Xizhijiang River Basin in Southern China was investigated.

OPTIMIZING MODEL

Objective Function
The objective of optimizing reservoir regulation is to increase the river discharge in the dry season and dry year so to satisfy the water demands in these periods. For this reason, the objective function of the optimization model is chosen to minimize the loss due to water supply deficits that can be described as:

\[ F = \min \sum_{t=1}^{T} f_t(x_t) \]  \hspace{1cm} (1)

Where \( t \) is the stage variable, \( T \) is the total stage of the studied period; \( x_t \) is the water supply deficit in stage \( t \) for the whole river basin users that is the difference of the water demand \( d_t \) and river discharge \( w_t \) at the water distribution node as following:

\[ x_t = \max(0, d_t - w_t) \]  \hspace{1cm} (2)

Where \( d_t \) is fixed, while \( w_t \) can be regulated by reservoir operation that can be expressed as following:

\[ w_t = s_t + q_t \]  \hspace{1cm} (3)

Where \( s_t \) is regional inflows between the reservoir dam and the distribution node, \( q_t \) is the reservoir discharge. \( f_t(x_t) \) is the loss function that is a function of \( x_t \), and is key to the model as different functions will result in different reservoir regulation strategies. An exact loss function is difficult to define, this paper proposed 3 different types of functions to study their impacts to reservoir operation schemes, which include linear function, parabolic function and exponential function.

**Linear function**
The function can be written as:

\[ f_t(x_t) = ax_t \]  \hspace{1cm} (4)

Where \( a \) is a parameter.
**Parabolic function**
The function can be written as:

$$f_t(x_t) = ax_t^2$$  \hspace{1cm} (6)

Where \(a\) is a parameter.

**Exponential function**
The function can be written as:

$$f_t(x_t) = e^{ax_t} - 1$$  \hspace{1cm} (5)

Where \(a\) is a parameter.

**Constraints**
The constraints include water balance constraint, flood control constraint, reservoir storage constraint, navigation constraint, turbine discharge constraint and power generation constraint that can be written as followings.

**Water balance constraint**

$$v_{t+1} = v_t + (r_t - q_t)\Delta t \hspace{1cm} t = 1,2,\ldots, T$$  \hspace{1cm} (7)

Where \(v_t\) is the reservoir storage in the beginning of stage \(t\), \(r_t\) is the average inflow of the reservoir in stage \(t\), \(q_t\) is the reservoir average discharge in stage \(t\), \(\Delta t\) is the time interval.

**Flood control constraint**

$$w_t \leq qf \hspace{1cm} t = 1,2,\ldots, T$$  \hspace{1cm} (8)

Where \(qf\) is the permitted maximum discharge at water distribution node for the flood control purpose.

**Reservoir storage constraint**

$$v_t \leq vm_t \hspace{1cm} t = 1,2,\ldots, T$$

$$v_t \geq vn_t \hspace{1cm} t = 1,2,\ldots, T$$  \hspace{1cm} (9)

Where \(vm_t\), \(vn_t\) are the permitted maximum and minimum reservoir storage at stage \(t\).
Navigation constraint
\[ w_t \geq q_n \quad t = 1, 2, \ldots, T \] (10)

Where \( q_n \) is the minimum river discharge at the distribution node to guarantee downstream navigation.

Turbine discharge constraint
\[ q_t \leq q_m \quad t = 1, 2, \ldots, T \] (11)

Where \( q_m \) is the permitted maximum discharge of the turbine for hydropower station.

Power generation constraint
\[ n_t \leq n_m \quad t = 1, 2, \ldots, T \] (12)

Where \( n_m \) is the maximum power that can be generated by the hydropower station, which is determined by water head applied to the turbine.

The optimization model
According to the above objective function and constraints, the optimization model can be written as followings.

\[
\begin{cases}
F = \min \sum_{t=1}^{T} f_t(x_t) \\
v_{t+1} = v_t + (r_t - q_t) \Delta t \\
w_t \leq q_f \\
q_n \leq w_t \leq q_m \\
0 \leq n_t \leq n_m \\
t = 1, 2, \ldots, T
\end{cases}
\] (13)

Model solution
The optimization model is a nonlinear optimization model, in this paper, the Dynamic Programming(DP) is employed to find the solution. The stage variable of DP is chosen as \( t \), \( v_t \) is chosen as state variable, while \( q_t \) is chosen as the decision variable. Due to the
page limitation, the detailed method for solving the optimization by using DP is not described here, which can be referred to Chen et al(1996)[2].

STUDY CASE

Xizhijiang River Basin
Xizhijiang River Basin (XRB) is chosen as the study case. XRB is located in Huidong County of Guangdong Province of Southern China, which is a tributary of East River with a drainage area of 4120km² and a length of 176km. There is a big reservoir, the Baipanzhou Reservoir, built in the upper stream of XRB, which has a storage capacity of 1.22 billion m³ and can regulates the river flow inter-annually. The original operational purposes of Baipanzhou Reservoir are mainly flood control and power generation.

XRB is the regional source river for water supply, it not only provides water for the regions along the river basin, but also supply water to the adjoining regions including Renping Peninsula, Huiyang County, Dayawan Economic Development District and Shenzhen City. Due to the regional economic development and population growth, the water demand increases sharply, which causes the water supply insufficient in the dry seasons and dry years. To optimal operate the Baipanzhou Reservoir to increase the river discharges in the dry seasons and dry years is a key issue for regional economic development.

The water distribution node of XRB is at Pinshan, which is 42 km down the Baipanzhou Reservoir dam site. Figure 1 is a sketch map of XRB up Pinshan.

Figure 1. Sketch map of Xizhijiang River Basin (XRB) up Pinshan
Water demand
In this paper, the water demand in the year of 2020 was used as case study. The projected water demands in 2020 in 12 months is listed in Table 1.

Table 1. Projected water demands in 2002                            unit:10^8 m^3

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Water demand</td>
<td>1.48</td>
<td>1.39</td>
<td>1.58</td>
<td>1.76</td>
<td>2.37</td>
<td>2.00</td>
</tr>
<tr>
<td>Water demand</td>
<td>2.42</td>
<td>2.00</td>
<td>1.58</td>
<td>1.44</td>
<td>1.39</td>
<td>1.39</td>
</tr>
</tbody>
</table>

RESULTS

Comparison of different objective functions
To test the effectiveness of different objective functions, the optimization model presented in this paper was employed to make the reservoir operation scheme. 45 year river inflows from 1955 to 2000 were used to make this investigation. Different lose functions have different reservoir operation schemes.

Results with linear function
With the linear objective function, the reservoir operation strategies and the water resources allocation schemes were derived. In this study, parameter a was tested using 6 values of 1, 5, 10, 20, 50, 100, and the results show that there are no differences among the water resources allocation schemes. This means the parameter a is not sensitive to the result, so in this paper, parameter a was taken the value of 1. Due to page limitation, the results were not listed here.

Results with parabolic function
With the parabolic objective function, the reservoir operation strategies and the water resources allocation schemes were derived. In the paper, parameter a was tested using 6 values of 1, 2, 5, 10, 50, 100, and the results show that there are no differences among the water resources allocation schemes, which means the parameter a is not sensitive to the result, so in this paper, parameter a was taken the value of 1. Due to page limitation, the results were not listed in this paper.

Results with exponential function
With the exponential objective function, the reservoir operation strategies and the water resources allocation schemes were derived. In this paper, parameter a was tested using 6 values of 1, 2, 5, 10, 50, 100, and the results show that with different value of a, the water resources allocation is different. Figure 2 shows the monthly water supply guaranteed ratio in 1987. From Figure 2, it can be seen that a bigger value of a will result in flat ratio fluctuation, that means water resources will be allocated evenly among the stages that is favorite by the planners. As the water resources scheme does not change when the value
of a is up 10, so in this paper, parameter a was taken the value of 10. Due to page limitation, the results were not listed in this paper.

Figure 2. Different results with different values of a for exponential lose function

Results comparison
The results show that with different lose functions, the water allocation schemes are different, to compare the effects, Figure 3 draws the water supply guaranteed ratio for three different water resources allocation schemes in 1987.

Figure 3. Water supply guaranteed ratio for different lose functions in 1987
Figure 3 show that the results with exponential lose function and parabolic lose function have the same results that have more flat monthly water supply guaranteed ratio.

Final results
According to the results comparison, the results with exponential function were chosen as the final results. Table 2 listed parts of the water resources allocation results.

CONCLUSION
This paper presents an optimization model for river basin water resources allocation based on reservoir regulation, and the Xizhijiang River Basin water resources allocation was investigated. The results show the model presented in this paper is reasonable.
Table 2. Final water resources allocation schemes

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan.</td>
<td>21.17</td>
<td>44.98</td>
<td>31.6</td>
<td>67.15</td>
<td>14.20</td>
<td>30.18</td>
</tr>
<tr>
<td>Feb.</td>
<td>17.01</td>
<td>38.58</td>
<td>23</td>
<td>52.16</td>
<td>10.83</td>
<td>24.57</td>
</tr>
<tr>
<td>Mar.</td>
<td>16.54</td>
<td>33.06</td>
<td>29.8</td>
<td>59.57</td>
<td>10.15</td>
<td>20.29</td>
</tr>
<tr>
<td>Apr.</td>
<td>53.15</td>
<td>94.99</td>
<td>52.9</td>
<td>94.55</td>
<td>39.88</td>
<td>71.27</td>
</tr>
<tr>
<td>May</td>
<td>75.22</td>
<td>100</td>
<td>75.22</td>
<td>100</td>
<td>54.54</td>
<td>72.51</td>
</tr>
<tr>
<td>June</td>
<td>63.36</td>
<td>100</td>
<td>63.36</td>
<td>100</td>
<td>63.36</td>
<td>100</td>
</tr>
<tr>
<td>July</td>
<td>73.22</td>
<td>95.45</td>
<td>73.2</td>
<td>95.43</td>
<td>73.47</td>
<td>95.78</td>
</tr>
<tr>
<td>Aug.</td>
<td>63.36</td>
<td>100</td>
<td>63.36</td>
<td>100</td>
<td>48.29</td>
<td>76.21</td>
</tr>
<tr>
<td>Sept.</td>
<td>50.02</td>
<td>100</td>
<td>50.02</td>
<td>100</td>
<td>47.37</td>
<td>94.70</td>
</tr>
<tr>
<td>Oct.</td>
<td>45.57</td>
<td>100</td>
<td>40.9</td>
<td>89.74</td>
<td>43.32</td>
<td>95.06</td>
</tr>
<tr>
<td>Nov.</td>
<td>41.42</td>
<td>93.94</td>
<td>31.5</td>
<td>71.44</td>
<td>40.37</td>
<td>91.55</td>
</tr>
<tr>
<td>Dec.</td>
<td>38.16</td>
<td>86.56</td>
<td>38.8</td>
<td>87.99</td>
<td>20.98</td>
<td>47.58</td>
</tr>
<tr>
<td>Total</td>
<td>558.2</td>
<td>73.96</td>
<td>573.66</td>
<td>84.84</td>
<td>466.76</td>
<td>68.31</td>
</tr>
</tbody>
</table>

* is the monthly water supply guaranteed ratio

REFERENCES


